On Biclop.na-continuous function in General Topology

1. A.P.Dhanabalan and 2. R.Buvaneswari

Department of Mathematics, Alagappa Govt. Arts College, Karaikudi-630 003,
Tamil Nadu, India.

E-mail: 1.danabalanap@yahoo.com
2.rambuviabi@yahoo.com

Abstract

The notion of biclop.na-continuous function has been introduced. Relationship between this new class of function with similar types of functions has been given. Some characterization and properties of such function are also being discussed.
2010 Mathematics Subject Classification: 54-XX.
Keywords: semi open (resp. semi closed), feebly open (resp. feebly closed), δ-open (resp. δ-closed), feebly clopen, δ-clopen, na-continuous, somewhat nearly open, feebly-T_2 (resp. feebly-regular, feebly-normal), δ-T_2 (resp. δ-regular, δ-normal), δ-connected.

1 Introduction:

In this paper, a unified version of some types of continuity has been introduced in topological space. We recall that in the subset A of X, the closure of A is the intersection of all closed sets containing A and the interior of A is the union of all open sets contained in A, denoted by cl(A) and int(A) respectively .

2 Preliminaries:

We recall the following definitions

Definition 2.1 *: A subset A of a topological space(X,τ)is said to be*
(i) semi open [5] if $A \subset cl(int(A))$
(ii) semi closed [5] if $int(cl(A)) \subset A$
(iii) feebly open[6] if $A \subset s\ cl(int(A))$
(iv) feebly closed[6] if $s\ int(cl(A)) \subset A$
(v) On the otherhand feebly open[4] if there exists an open set U of X such that $U \subset A \subset s\ cl(U)$*, where s cl(U) denotes the semi-closure of U*
(vi) δ*-open [9] if for each* $x \in A$*, there exists an open set G such that* $x \in G \subseteq int(cl(G)) \subseteq A$
(vii) on the other hand δ*-open[9] if for each* $x \in A$*, there exists a regular open set U of X such that* $x \in U \subset A$
(viii) feebly clopen[2] if it is both feebly open and feebly closed
(ix)δ*-clopen if it is both* δ*-open and* δ*-closed*
(x) somewhat nearly open [7] if $int(cl(A)) \neq \varphi$

Remark 2.2 *:Let A be a subset of (X,τ)*
(i) The feebly closure of A is the intersection of all feebly closed set containing A and denoted by f.cl(A)
(ii) the feebly interior of A is the union of all feebly open sets contained in A and is denoted by f.int(A).

Definition 2.3 *: A map f:X → Y is said to be*
(i) feebly open [8] if the image of each open set in X is feebly open in Y

(ii) feebly closed [8] if the image of each closed set in X is feebly closedm in Y
(iii) feebly continuous [8] if the inverse image of each open set V of Y is feebly open in X
(iv) feebly clopen [2] if the image of every open and closed set in X is feebly open and feebly closed in Y
(v) δ-clopen if the image of every open and closed set in X is δ-open and δ-closed in Y
(vi) δ-open if the image of each open set in X is δ-open in Y
(vii) δ-closed if the image of each closed set in X is δ-closed in Y
(viii) na-continuous [3] if the inverse image of every feebly open set A of Y is δ-open in X
(ix) δ-irresolute if the inverse image of every δ-open set A of Y is δ-open

Remark 2.4 *:(i) Every open set is feebly open [1]*
(ii) Every closed set is feebly closed [1]
(iii) Every feebly clopen is semi clopen at the diagram in [2]

3 The Main results

:In this section, the new function of biclop.na-continuous is introduced and also formed the related functions and discussed among them.

Definition 3.1 *:A function f:X → Y is said to be biclop.na-continuous if the inverse image of every feebly clopen set V of Y is δ-clopen in X.*

Example 3.2 *:Let* $X = \{u,v,w\}$ *be a topological space with topology* $\tau = \{ \varphi, X, \{v\}, \{u,w\} \}$
let $Y = \{ p,q,r\}$ *with topology* $\sigma = \{ \varphi, Y, \{q\}, \{p,r\} \}$
Clearly τ is a δ- clopen set in X and σ is a feebly clopen set in Y
Define f: X→Y such that f(u) =r, f(v) =q, f(w)=p
Clearly the inverse image of feebly clopen set in Y is δ- clopen set in X and so f is biclop.na-continuous.
Result: (i) f is open map and closed map then f is clopen map.
(ii) f is continuous.
(iii) From (i) and (ii) we get f is biclop.na-continuous.
f is homeomorphism since we know that fis open, continuous and bijective.

Definition 3.3 *:Let A be subset of (X,τ). A is said to be somewhat nearly clopen if int (cl(A))≠ φ and cl(int(A))≠ φ that implies that cl(int(cl(A)))≠ φ.*

Definition 3.4 *:A function f:X → Y is somewhat nearly biclop.na-continuous if* $f^{-1}(V)$ *is somewhat nearly clopen for every feebly clopen set V in Y such that* $f^{-}(V) \neq \varphi$.

Remark 3.5 *:Somewhat nearly clopen set is somewhat nearly open and somewhat nearly closed.*

Definition 3.6 *:A function f: X → Yis slightly biclop.na-continuous if for each x ϵ X and each feebly clopen set V of Y containing f(x) there exist a δ-open set U containing x such that f(U)⊆ V.*

Definition 3.7 *:A function f:X → Y is almost biclop.na-continuous if for each x ϵ X and each feebly clopen set V of Y containing f(x) there exist a δ - clopen set U containing x such that f(U)⊆ int (cl (V)).*

Definition 3.8 *:A function f:X →Y is δ-clopen irresolute if the inverse image of every δ - clopen subset in Y is δ-clopen subset in Y is δ-clopen in X.*

Theorem 3.9 *:Every biclop.na-continuous is na-continuous.*
Proof:Let f:X →Y be biclop.na-continuous mapping. To show that f is na-continuous, let H be anyfeebly clopen subset of Y. Since f is biclop.na-continuous, then $f^{-}(H)$ *is δ-clopen in X, and so 2.1 (ix),* $f^{-}(H)$*is δ-open in X. Hence f is na-continuous map.*

Theorem 3.10 *:For a function f:X $\to$ Y, the following statements are equivalent.*
(a) f is biclop.na-continuous
(b) A function f:X $\to$ Y is biclop.na-continuous if for every feebly clopen set V of Y containing f(x) there exist δ - clopen set U containing x such that f(U)$\subseteq$ V.
Proof:(a)$\Rightarrow$(b): Let xϵX and let V be a feebly clopen set in Y containing f(x). Then, by (b),$f^{-1}(V)$ is δ-clopen in X containing x. Let U=$f^{-1}(V)$. Then, f(U)$\subseteq$ V.
(b)$\Rightarrow$(a): Let V be a feebly clopen set of Y, and let xϵ $f^{-1}(V)$. Since f(x)ϵV, there exists U containing x such that f(U)$\subseteq$V. It then follows that xϵU $\subseteq f^{-1}(V)$. Hence $f^{-1}(V)$ is δ-clopen.

Theorem 3.11 *: If f:X$\to$Y is biclop.na-continuous, then it is slightly biclop.na-continuous.*
Proof: Let xϵX, and let Vis feebly clopen. Since f is biclop.na-continuous, there exist δ-clopen set U containing x such that f(U)$\subseteq$V. Since δ - clopen set is δ - open, we have U is δ - open and so f is slightly biclop.na-continuous. Thus f is slightly biclop.na-continuous.

Theorem 3.12 *: If f is a mapping of X into Y and X=$X_1 \cup X_2$, where X_1 and X_2 are δ-clopen set and f/X_1 and f/X_2 are biclop.na-continuous, then f is biclop.na-continuous.*
Proof: Let A be a feebly clopen subset of Y. Then, since (f/X_1) and (f/X_2) are both biclop.na-continuous, therefore $(f/X_1)^{-1}(A)$ and $(f/X_2)^{-1}(A)$ are both δ-clopen set in X_1 and X_2 respectively. Since X_1 and X_2 are δ-clopen subsets of X, therefore $(f/X_1)^{-1}(A)$ and $(f/X_2)^{-1}(A)$ are both δ-clopen subsets of X. Also, $f^{-1}(A) = (f/X_1)^{-1}(A) \cup (f/X_2)^{-1}(A)$. Thus $f^{-1}(A)$ is the union of two δ-clopen sets and is therefore δ-clopen. Hence f is biclop.na-continuous.

Theorem 3.13 *:If f is a mapping of X into Y and X=$X_1 \cup X_2$ and if (f/X_1) and (f/X_2) are both biclop.na-continuous at a point x belongs to $X_1 \cap X_2$, then f is biclop.na-continuous at x.*
Proof: Let U be any feebly clopen set containing f(x). Since xϵ $X_1 \cap X_2$ and (f/X_1), (f/X_2) are both biclop.na-continuous at x, therefore there exist δ-clopen sets V_1 and V_2 such that xϵ $X_1 \cap V_1$ and f($X_1 \cap V_1$) $\subset$ U, and xϵ $X_2 \cap V_2$) and f($X_2 \cap V_2$) $\subset$ U. Now since X=$X_1 \cup X_2$, therefore f($V_1 \cap V_2$)=f($X_1 \cap V_1 \cap V_2$)$\cup$f($X_2 \cap V_1 \cap V_2$) $\subset$ f($X_1 \cap V_1$) $\cup$ f($X_2 \cap V_2$)$\subset$ U. Thus,$V_1 \cap V_2$=V is a δ-clopen set containing x such that f(V)$\subset$ U and hence f is biclop.na-continuous ar x.

Theorem 3.14 *: Every restriction of a biclop.na-continuous mapping is biclop.na-continuous.*
Proof:Let f be a biclop.na-continuous mapping of X into Y and let A be any subset of X. For any feebly clopen subset S of Y, $(f/A)^{-1}(V)$=A$\cap f^{-1}(V)$. But f being biclop.na-continuous, $f^{-1}(S)$ is δ-clopen and hence A$\cap f^{-1}(V)$ is a relatively δ-clopen subset of A, that is $(f/A)^{-1}(V)$ is a δ-clopen subset of A. Hence f/A is biclop.na-continuous.

Theorem 3.15 *: Let f map X into Y and let x be a point of X. If there exist a δ-clopen set N of x such that the restriction of f to N is biclop.na-continuous at x, then f is biclop.na-continuous at x.*
Proof: Let U be any feebly clopen set containing f(x). Since f/N is biclop.na-continuous at x, therefore there is an δ-clopen set V_1 such that xϵN $\cap V_1$ and f(N $\cap V_1$) $\subset$ U. Thus N $\cap V_1$ is δ-clopen set of x.

Theorem 3.16 *: Let X=$R_1 \cup R_2$, where R_1 and R_2 are δ-clopen sets in X. Let f:$R_1 \to$Y and g:$R_2 \to$Y be biclop.na-continuous. If f(x)=g(x) for each x$\epsilon R_1 \cap R_2$. Then h:$R_1 \cap R_2 \to$Y such that h(x)=f(x) for xϵR_1 and h(x)=g(x) for xϵR_2 is biclop.na-continuous.*
Proof: Let U be a feebly clopen set of Y. Now $h^{-1}(U)$=$f^{-1}(U) \cup g^{-1}(U)$. Since f and g are biclop.na-continuous, $f^{-1}(U)$ and $g^{-1}(U)$ are δ-clopen set in R_1 and R_2 respectively. But R_1 and R_2 are both δ-clopen sets in X. Since union of two δ-clopen sets is δ-clopen, so $h^{-1}(U)$ is a δ-clopen set in X. Henxe h is biclop.na-continuous.

Theorem 3.17 *: Let f:X $\to$Y be biclop.na-continuous surjection and A be δ-clopen subset of X. If f is feebly clopen function, then the function g:A $\to$f(A), defined by g(x)=f(x) for each xϵA is biclop.na-continuous.*
Proof: Suppose that H=f(A). Let xϵA and V be any feebly clopen set in H containing g(x). Since H is feebly clopen set in Y and V is feebly clopen in H. Since f is biclop.na-continuous, hence there exist a δ-clopen set U in X containing x. Taking W=U$\cap$A, since A is δ-open and δ-closed subset of X, W is a δ-clopen set in A containing x. Thus g is biclop.na-continuous.

Theorem 3.18 *: Let f:X $\rightarrow$Y be biclop.na-continuous. If Y is a semi clopen subset of Z, then f: X $\rightarrow$Z is biclop.na-continuous.*

Proof: Let V be any feebly clopen set of Z. Since Y is semi clopen, V$\cap$Y is a semi clopen set in Y. Since f is biclop.na-continuous, $f^{-1}(V \cap Y)$ is δ-clopen in X. But f(x)ϵY for each xϵX. Thus $f^{-1}(V) = f^{-1}(V \cap Y)$ is a δ-clopen set of X. Therefore f:X $\rightarrow$Z is biclop.na continuous.

Theorem 3.19 *:Let f:X $\rightarrow$Y and g:Y $\rightarrow$Z be function. Then the composition function g$\circ$f is δ-irresolute if f is na-continuous and g is almost open and it is δ-clopen irresolute where f is biclop.na-continuous and g is almost biclop.na-continuous.*

Proof: To Prove: g$\circ$f is δ-irresolute. Given f is na-continuous and g is almost open. Let W be any δ-open subset of Z. Since g is almost open, so $g^{-1}(W)$ is open subset of Y. Since f is na-continuous, by **??***, $(g \circ f)^{-1}(W)$=$f^{-1}(g^{-1}(W))$ is δ-open subset in X. Thus g$\circ$f is δ-irresolute function.*

To Prove: g$\circ$f is δ-clopen irresolute. Given f is biclop.na-continuous and g is almost clopen continuous. Let U be any δ-clopen subset of Z. Since g is almost clopen continuous, so $g^{-1}(U)$ is clopen subset of Y. Since f is biclop.na-continuous, by **??***. Now $(g \circ f)^{-1}(U) = f^{-1}(g^{-1}(U))$ is δ-clopen subset in X. Thus g$\circ$f is δ-clopen irresolute.*

Theorem 3.20 *: Let f:X $\rightarrow$Y and g:Y $\rightarrow$Z be two mappings such that their composition g$\circ$f:X $\rightarrow$Z be biclop.na-continuous then the following statement true. If f is δ-clopen irresolute and surjective, then g is biclop.na-continuous.*

Proof: Let A be δ-clopen set of Y. Since f is δ-clopen irresolute, $f^{-1}(A)$ is δ-clopen in X. Since $(g \circ f)(f^{-1}(A))$ is feebly-clopen in Z. That is g(A) is feebly-clopen in Z, since f is surjective. Therefore g is biclop.na-continuous.

4 Some properties of na-continuous function

Remark 4.1 *: A function f:X $\rightarrow$Y is na-continuous if for every feebly open set V of Y containing f(x) there exist δ-open set U containing x such that f(U)$\subseteq$V.*

Definition 4.2 *: A topological space X is said to be*

(i) Feebly-T_2 or Feebly Hausdorff if for any pair of distinct points x,y of X, there exists disjoint feebly open sets U and V such that x ϵ U and y ϵ V.

(ii) Feebly-regular if each feebly closed set F and each point x $\notin$ F, there exist disjoint feebly-open sets U and V such that x ϵ U, F$\subseteq$ V.

(iii) Feebly-normal if for any two disjoint feebly closed subsets F and K, there exist disjoint feebly-open sets U and V such that F$\subseteq$U, K$\subseteq$V.

Definition 4.3 *: A topological space X is said to be*

(i) δ-T_2 if for any two distinct points x,y of X there exist two disjoint δ-open sets U and V containing x and y respectively.

(ii) δ-regular if for each δ-closed set F and each point x$\notin$F, there exist disjoint δ-open sets U and V such that xϵU, F$\subseteq$V.

(iii) δ-normal if for any two disjoint δ-closed subsets F and K, there exist disjoint δ-open sets U and V such that F$\subseteq$U, K$\subseteq$V.

Theorem 4.4 *: If f:X$\rightarrow$Y is na-continuous injection and Y is feebly-T_2, then X is δ-T_2.*

Proof:Let x and y be two distinct points of X. Then f(x) and f(Y) are two distinct points of Y. Thus, there exist two disjoint feebly open U and V containing x and y respectively. Then by remak4.1, $f^{-1}(U)$ and $f^{-1}(V)$ are two δ-open sets in X. Clearly, x$\epsilon f^{-1}(U)$, y$\epsilon f^{-1}(V)$, and $f^{-1}(U) \cap f^{-1}(V)$=φ. Thus X is μ-T_2. Hence proved.

Theorem 4.5 *: If f:X$\rightarrow$Y is na-continuous, bijection and X is δ-regular, then Y is feebly-regular.*

Proof: Let F be a feebly closed subset of Y and let y$\notin$F. Let y=f(x). since f is na-continuous, by remark4.1, $f^{-1}(F)$ is δ-closed in X so that $f^{-1}(Y)$=x$\notin$ $f^{-1}(F)$. Let G=$f^{-1}(F)$. Then x$\notin$G; thus, by δ-regularity of

X, there exist two disjoint δ-open sets U and V such that G⊆U and x∊V. Thus, we have F=f(G)⊆f(U) and Y=f(x)∊f(V) and f(U)∩f(V)=φ. As f is na-continuous, f(U) and f(V) are feebly open in Y. Thus f(U)∩f(V)=φ, y=f(x)∊f(V) and F⊆f(U). Hence Y is feebly-regularity.

Theorem 4.6 *: If f:X→Y is na-continuous, surjection, and X is δ-normal then Y is feebly-normal.*
Proof: Let F_1 and F_2 be two disjoint feebly closed subsets of Y. Since f is na-continuous, by remark 4.1, $f^{-1}(F_1)$ *and* $f^{-1}(F_2)$ *are two δ-closed subsets in X. Let* $K_1=f^{-1}(F_1)$, *and let* $K_2 = f^{-1}(F_2)$. *Then* K_1 *and* K_2 *are two disjoint δ-closed subsets of (X,μ). Since X is δ-normal, there exists two disjoint δ-open sets U and V such that* $K_1 \subseteq U$ *and* $K_2 \subseteq V$. *We thus have* $F_1=f(K_1)\subseteq f(U)$ *and* $F_2=f(K_2)\subseteq f(V)$. *Also f(U) and f(V) are two disjoint feebly open sets in Y. Hence Y is feebly-normal.*

Definition 4.7 *: A topological space X is said to be δ-connected if X cannot be written as union of two non empty δ-open sets.*

Theorem 4.8 *: If f:X→Y is na-continuous surjection and X is δ-connected, then Y is connected.*
Proof: Let us assume that Y be not connected. Then, there exist non empty disjoint feebly open V_1 *and* V_2 *such that* $Y=V_1\cup V_2$. *Hence, we have* $f^{-1}(V_1)\cup f^{-1}(V_2)=X$ *and* $f^{-1}(V_1)\cap f^{-1}(V_2)=\varphi$. *Since f is surjective,* $f^{-1}(V_1)$ *and* $f^{-1}(V_2)$ *are non-empty. Since* V_1 *and* V_2 *are feebly open in Y. Thus by remark4.1,* $f^{-1}(V_1)$ *and* $f^{-1}(V_2)$ *are δ-open. Therefore X is not δ-connected. This is a contradiction.*

References

[1] K. Ali, *On semi separation axioms*, M.Sc.Thesis Al.Mastaxi riyah Univ., 2003.

[2] A.P. Dhana Balan and R. Buvaneswari, *Totally feebly continuousfunctions*, Mathematical Sciences International Research Journals, 3, 2, 2014.

[3] G.I. Chae, T. Noiri and D.W. Lee, Kyungapook Mash. J. 26, 73-79, 1986.

[4] S.G. Crossely and S.K. Hildebrand Texas J.Sci. 23-26, 1971.

[5] N. Levine, AM.Math. Monthly 70, 36-41,1963.

[6] S.N. Maheswari and U. Tapi, *Note on some applications on feebly open sets*, M.B.J.Univ. Saugar, 1979.

[7] Z. Piotrocoski, *A survey of results concerning generalized continuity in topological spaces*, Acta Math. Univ. Comexian, 91-110, 1987.

[8] V. Popa, Glasnik Math. III 14(34):359-362.

[9] N.V. Velicko, *H-closed topological spaces*, Amer Math. Soc. Transl., 78,103-118, 1968.